Two Dimensional Hydrodynamic and Sediment Transport Model for Surface Flow Routing (CHRE2D-UA licensed)

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Background

- There is no defined drainage path in urban watershed resided in alluvial fan.
- Modeling surface flow over watershed needs to simulate both overland flow and channel flow.
- The depth of overland flow is much shallower than that of channel flow.

Natural Alluvial Fan

Urbanized Alluvial Fan - Reno
Computational Hydraulics and River Engineering Laboratory (CHRE2D)

- CHRE2D is a two-dimensional Hydrodynamic and Sediment transport model that simulates surface flow routing and sediment transport using numerical solutions of shallow water equations and the kinematic or diffusion wave approximation.

- The shallow water equations are discretized by the first-order Godunov-type finite volume method. An approximate solution to the momentum equation, kinematic or diffusion wave approximation, was introduced to overcome the difficulties in simulating very shallow overland (e.g. cm).

- The resulted CHRE2D model is capable of simulating both hydrological flow (e.g. surface flow routing) and hydraulic flow (e.g. dam break), which has not been achieved in similar commercial software, such as FLO2D, ARM2D.

- Additionally, the CHRE2D model implemented the Grass-type sediment transport formula to simulate the total sediment load in both overland flow and channel flow.
Methodology

- Governing Equation for Flow Simulation

Shallow water equations: mass conservation and momentum equations

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = i_0
\]

\[
\begin{align*}
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2 + \frac{1}{2}gh^2)}{\partial x} + \frac{\partial (huv)}{\partial y} &= ghS_{0x} - C_f \|u\|u \\
\frac{\partial (hv)}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2 + \frac{1}{2}gh^2)}{\partial y} &= ghS_{0y} - C_f \|u\|v
\end{align*}
\]
SWEs in Vector Form

\[ \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S_0 - S_f + S_r \]

\[ Q = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix} \quad F = \begin{pmatrix} hu \\ hu^2 + gh^2 / 2 \\ huv \end{pmatrix} \quad G = \begin{pmatrix} hv \\ huv \\ hv^2 + gh^2 / 2 \end{pmatrix} \]

\[ S_0 = \begin{pmatrix} 0 \\ ghS_{0x} \\ ghS_{0y} \end{pmatrix} \quad S_f = \begin{pmatrix} 0 \\ C_f \|u\|u \\ C_f \|u\|v \end{pmatrix} \quad S_r = \begin{pmatrix} i_0 \\ 0 \\ 0 \end{pmatrix} \]
The integral form of the shallow water equations is:

\[
\frac{\partial}{\partial t} \int_{\Omega} Q d\Omega + \int_{\Gamma} F d\Gamma = \int_{\Omega} (S_0 + S_f + S_r) d\Omega
\]

The discretized form of the governing equations can be expressed as

\[
Q^{(n+1)} = Q^{(n)} - \frac{\Delta t}{\Delta A} \sum_{k=1}^{4} F_k (Q_L^{(n)}, Q_R^{(n)}) \cdot n_k L_k + \Delta t (S_0^{(n)} + S_f^{(n)} + S_r^{(n)})
\]

\(Q_L\) and \(Q_R\) are the reconstructed conservative variables at the left and right side of the edge \(k\), \(n_k\) is the normal vector, and \(L_k\) is the length of edge.
Numerical Method

The HLL Riemann solver is used in the model to evaluate the advective flux across cell interface. The advective flux is calculated by:

\[
F(Q_L, Q_R) = \begin{cases} 
  F(Q_L), & \text{if } s_L \geq 0 \\
  F^*(Q_L, Q_R), & \text{if } s_L < 0 < s_R \\
  F(Q_R), & \text{if } s_R \leq 0 
\end{cases}
\]

The flux at the star region is determined by

\[
F^*(Q_L, Q_R) = \frac{s_R F(Q_L) - s_L F(Q_R) + s_L s_R (Q_R - Q_L)}{s_R - s_L}
\]
Wet and Dry Bed

For wet cells, wave speed is calculated as

\[
\begin{align*}
    s_L &= \min(u_L^n - \sqrt{gh_L}, u_s - c_s) \\
    s_R &= \max(u_R^n + \sqrt{gh_R}, u_s + c_s)
\end{align*}
\]

and

\[
\begin{align*}
    u_s &= \frac{(u_L^n + u_R^n)}{2} + \left(\sqrt{gh_L} - \sqrt{gh_R}\right) \\
    c_s &= \frac{(u_L^n - u_R^n)}{4} + \left(\sqrt{gh_L} + \sqrt{gh_R}\right) / 2
\end{align*}
\]

For the dry bed situation, the estimated wave speeds are replaced by the exact dry front speed:

right dry bed: \[
\begin{align*}
    s_L &= u_L^n - \sqrt{gh_L} \\
    s_R &= u_R^n + 2\sqrt{gh_R}
\end{align*}
\]

left dry bed: \[
\begin{align*}
    s_L &= u_L^n - 2\sqrt{gh_L} \\
    s_R &= u_R^n + \sqrt{gh_R}
\end{align*}
\]

The numerical model described here is a Godunov-type finite volume method of first-order accuracy in both space and time. The stability of the model is constrained by the

\[
CFL = \frac{\Delta t}{\Delta x} (\|u\| + \sqrt{gh})
\]
Diffusion Wave Approximation

When the flow depth is less than a threshold value ($10^{-5} - 10^{-10}$ m), the flow velocities are updated implicitly by using the diffusion wave approximation

$$u^{(n+1)} = \begin{cases} 
  + \frac{1}{n} \frac{\sqrt{-\eta_x}}{(1 + (\eta_y / \eta_x)^2)^{1/4}} (h^{(n+1)})^{2/3}, & \text{if } \eta_x < 0 \\
  0, & \text{if } \eta_x = 0 \\
  - \frac{1}{n} \frac{\sqrt{\eta_x}}{(1 + (\eta_y / \eta_x)^2)^{1/4}} (h^{(n+1)})^{2/3}, & \text{if } \eta_x > 0 
\end{cases}$$

$$v^{(n+1)} = \begin{cases} 
  + \frac{1}{n} \frac{\sqrt{-\eta_y}}{(1 + (\eta_x / \eta_y)^2)^{1/4}} (h^{(n+1)})^{2/3}, & \text{if } \eta_y < 0 \\
  0, & \text{if } \eta_y = 0 \\
  - \frac{1}{n} \frac{\sqrt{\eta_y}}{(1 + (\eta_x / \eta_y)^2)^{1/4}} (h^{(n+1)})^{2/3}, & \text{if } \eta_y > 0 
\end{cases}$$

where $\eta_x$ and $\eta_y$ are surface gradients, $n$ is Manning's roughness.
Variable Density Flow Model

The governing equations for variable density flow model is based on the two-phase flow theory and treated sediment-laden flow density as a spatial and temporal variable.

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = S_b
\]

\[
\frac{\partial (\rho h)}{\partial t} + \frac{\partial (\rho hu)}{\partial x} + \frac{\partial (\rho hv)}{\partial y} = \rho_b S_b
\]

\[
\frac{\partial (\rho hu)}{\partial t} + \frac{\partial (\rho huu + \frac{1}{2} \rho gh^2)}{\partial x} + \frac{\partial (\rho huv)}{\partial y} = \rho ghS_{0x} - \rho C_f \|u\| u
\]

\[
\frac{\partial (\rho hv)}{\partial t} + \frac{\partial (\rho hvu)}{\partial x} + \frac{\partial (\rho hvv + \frac{1}{2} \rho gh^2)}{\partial y} = \rho ghS_{0y} - \rho C_f \|u\| v
\]
Vector Form of Variable Density Equations

\[
\frac{\partial Q}{\partial t} + \frac{\partial F_x(Q)}{\partial x} + \frac{\partial F_y(Q)}{\partial y} = S_0(U) - S_f(U) + S_b(U)
\]

\[
U = \begin{pmatrix} h \\ \rho \\ u \\ v \end{pmatrix}, \quad Q = \begin{pmatrix} h \\ \rho h \\ \rho hu \\ \rho hv \end{pmatrix}, \quad F_x(Q) = \begin{pmatrix} hu \\ \rho hu \\ \rho hu + \frac{1}{2} \rho gh^2 \\ \rho hv \end{pmatrix}, \quad F_y(Q) = \begin{pmatrix} hv \\ \rho hv \\ \rho hv + \frac{1}{2} \rho gh^2 \end{pmatrix}
\]

\[
S_0(U) = \begin{pmatrix} 0 \\ 0 \\ \rho gh S_{0x} \\ \rho gh S_{0y} \end{pmatrix}, \quad S_f(U) = \begin{pmatrix} 0 \\ 0 \\ C_f \|u\|u \\ C_f \|u\|v \end{pmatrix}, \quad S_b(U) = \begin{pmatrix} S_b \\ \rho_b S_b \\ 0 \\ 0 \end{pmatrix}
\]
Sediment Transport Model

The evolution of mobile bed is described by the following conservation equation of bed material:

\[
\frac{\partial b}{\partial t} = -S_b
\]

The bed material flux between flow and mobile bed is evaluated by the non-equilibrium sediment transport formula. The formula reads:

\[
S_b = \frac{1}{1 - \phi} \frac{(q_b - q_b^*)}{L}
\]

The calculation of adaption length is

\[
L = \max \left( L_b, \frac{h \|u\|}{\alpha_0 \omega_0} \right)
\]
Sediment Transport Model

The settling velocity of sediment particle is calculated by

\[ \omega_0 = \sqrt{(13.95 \nu / d_{50})^2 + 1.09 sgd_{50} - 13.95 \nu / d_{50}} \]

The sediment transport capacity is calculated by the modified Meyer-Peter-Müller (MPM) formula:

\[ q^*_b = 12\sqrt{sgd_{50}^3 (\theta - \theta_c)^{1.5}} \]

where

- \( \theta = u^*_2 / sgd \) = Shields number
- \( u_\ast = C_f^{1/2} \|u\| \) = friction velocity
- \( \theta_c = 0.047 \) = critical Shields number
Numerical Method

The integral form of the variable density shallow water equations is:

\[
\frac{\partial}{\partial t} \int_{\Omega} Q d\Omega + \int_{\partial \Omega} (Fn) d\partial\Omega = \int_{\Omega} S_0 d\Omega - \int_{\Omega} S_f d\Omega + \int_{\Omega} S_b d\Omega
\]

The first-order, two-step fractional scheme is employed in the numerical model to update the solution at each time step:

**Step #1:**

\[
Q^{(*)} = Q^{(n)} - \frac{\Delta t}{A} \sum_{k=1}^{4} F_k (Q_L^{(n)}, Q_R^{(n)}) n_k l_k + \Delta t (S_0^{(n)} - S_f^{(n)})
\]

**Step #2:**

\[
\begin{align*}
Q^{(n+1)} &= Q^{(*)} + \Delta t S_b^{(*)} \\
\left\{ b^{(n+1)} = b^{(n)} - \Delta t \frac{S_b^{(*)}}{1 - \phi} \right. \\
\end{align*}
\]
Numerical Method

The HLLC approximate Riemann solver is extended to approximate numerical fluxes across cell boundaries. The advective flux is calculated by:

\[
F_{HLLC}(Q_L, Q_R) = \begin{cases} 
F_L & 0 \leq S_L \\
F_{*L} & S_L < 0 \leq S_* \\
F_{*R} & S_* < 0 \leq S_R \\
F_R & S_R < 0 
\end{cases}
\]

The flux at the star region is determined by

\[
Q_{*L} = h_L \frac{U_L - S_L}{S_* - S_L} \begin{pmatrix} 1 \\ \rho_L \\ \rho_L (S_* n_x + u_L n_y) \end{pmatrix} \\
Q_{*R} = h_R \frac{U_R - S_R}{S_* - S_R} \begin{pmatrix} 1 \\ \rho_R \\ \rho_R (S_* n_x + u_R n_y) \end{pmatrix}
\]

\[
F_{*L} = F_L + S_L (Q_{*L} - Q_L) \\
F_{*R} = F_R + S_R (Q_{*R} - Q_R)
\]
Test Case 1
Goodwin Creek Experimental Watershed

- The Goodwin Creek Experimental Watershed resides in the Panola County, Mississippi. As a tributary of Long Creek, it flows into the Yocona River, Yazoo River Basin.
- The watershed drainage area is 21.3 km$^2$. The watershed elevation ranges from 71 m to 128 m above the sea level. The DEM resolution used in the test is 30 m by 30 m.
- The rainfall event of Oct. 17, 1981, was simulated. Soil type, land use, precipitation and DEM data was based on published NRCS report.
Simulated Results
Flow Depth

G-2

G-3

G-4

G-5

G-6

G-7

Measured
SWE-KWA

Time (s)
Flow Depth (m)
Test Case 2: Tucson, Arizona, July 27th to Aug. 4th, 2006 Event – Santa Cruz Basin (62 miles x 24 miles)
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Past Event – Flow Discharge and Depth

Tucson Map (Google Maps API)

Weather Station
- AFRS
- RAWS
- NWS

Forecast data
- DEM
- Flow Depth
- Flow Velocity

Hyetograph
- Region: 18599
- Select

Flow Discharge

Flow Depth
Past Event – Street View

T = 31 hrs
Test #3: State of Arizona

Hypothetical Case: 1.0 inch precipitation over the State of Arizona for a few hours (10 hrs)
Test Case 3: State of Arizona
Hypothetical 1 inch precipitation

T = 6 hrs
T = 12 hrs
T = 20 hrs
Test Case 3: State of Arizona
Phoenix Area

T = 6 hrs
T = 12 hrs
T = 20 hrs
Test Case 4: 1996 Lake Ha! Ha! Catastrophic Flood Event

- The 1996 Lake Ha! Ha! catastrophic flood event occurred in the Saguenay region of Quebec, Canada.
- From July 18 to 21, 1996, an extreme precipitation event affected the Saguenay region of Quebec, Canada. At the Ha! Ha! Lake, an earthfill dyke was being overtopped by up to 0.26 m of water, and a new outlet channel formed.
- The failure of the dyke resulted in a peak discharge of 8 times the 100-year flood. The Ha! Ha! River was severely damaged by the resulting flood flow \[\textit{Brooks and Lawrence, 1999}\].
Simulation Domain

- The numerical simulation started with the digital elevation model (DEM) of the Ha! Ha! River, which was surveyed in May 1994 [Capart et al., 2007]. The spatial data is based on the Modified Transverse Mercator (MTM) projection, zone 7 coordinates (NAD83).

- The DEM data: $275,000 < x < 282,000$ m on the east-west direction, $5,318,000 < y < 5,354,000$ m in north-south direction.

- The numerical simulation started with the digital elevation model (DEM) of the Ha! Ha! River, which was surveyed in May 1994 [Capart et al., 2007]. The spatial data is based on the Modified Transverse Mercator (MTM) projection, zone 7 coordinates (NAD83).
Bed Elevation Changes

Selected Cross Section Changes
Logitudinal Profile

Fig 16. Measured and calculated thalwegs of test case 4: (a) 0 – 12 km; (b) 12 – 24 km; (c) 24 – 36 km.
Conclusions

- CHRE2D model is a robust surface flow routing and sediment transport model, which is capable of simulating hydrodynamics of unsteady flow, surface flow over watershed, and sediment transport processes.
- The performance of the model was verified by many laboratory and field cases.
- For flow simulation, the model predicted accurately peak flows and flow hydrographs.
- The sediment module predicted reasonable changes of river cross sections and thalweg caused by a realistic dam break flow.
- The accuracy and simplicity of the proposed model, together with the robust implementation of well-balanced numerical scheme, makes this model suitable for practical hydraulic engineering applications.
- CHRE2D is licensed by the University of Arizona.
References:


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Thank you!

Questions?